Economics 704a Lecture 6: New Keynesian Model

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New Keynesian Model: Outline

New Keynesian Model: Roadmap

- The textbook New Keynesian model includes:
    1. Money (in the utility function)
    2. Monopolistic Competition
    3. Nominal Rigidities
    - NO Capital (nominal bonds in zero net supply)
- We have been introducing these ingredients one by one.
- Last class, discussed simple, one-period nominal rigidity which gives some intuition.
    - Price is markup over *expected* marginal cost.
    - When marginal cost rises, markups fall. This happens with a monetary expansion.
    - Technology shocks are contractionary because aggregate demand is predetermined.
- Today, add more persistent price stickiness and get most of way to three equation model.

New Keynesian Model: Roadmap

- Three "blocks" to the model plus a central bank rule:

1. Household: Same as in money model.
    - Optimality conditions generate "Dynamic IS" curve that gives relationship between output and real interest rate.
    - Intertemporal substitution along Euler combined with $Y = C$.
    - Note: Different from Old Keynesian IS, which was investment response to r + strong Keynesian multiplier
2. Firms: Same as imperfect competition model, with addition of persistent nominal rigidity for intermediate producers.
    - Generates a "New Keynesian Phillips Curve,"
      a forward-looking, expectations-augmented Phillips curve.
3. Monetary authority's nominal interest rate rule closes model.
    - Taylor (1993) was a *descriptive* paper about how Volcker and Greenspan had conducted monetary policy.
    - Clarida, Gali, and Gertler added to NK model as a CB rule to close model.

Household Problem

$$\max_{C_t,N_t,B_t,M_t} E_t \left\{ \sum_{s=0}^{\infty} \beta^s \left(\frac{C_{t+s}^{1-\gamma}}{1-\gamma} + \zeta \frac{(M_{t+s}/P_{t+s})^{1-\nu}}{1-\nu} - \chi \frac{N_{t+s}^{1+\varphi}}{1+\varphi} \right) \right\} \text{ s.t.}$$

$$C_t = \frac{W_t}{P_t} N_t - \frac{B_t - Q_{t-1} B_{t-1}}{P_t} - \frac{M_t - M_{t-1}}{P_t} + TR_t + PR_t$$

- FOCs :

$$\frac{W_t}{P_t} = \frac{\chi N_t^{\varphi}}{C_t^{-\gamma}} \tag{1}$$

$$\frac{M_t}{P_t} = \zeta^{1/\nu} \left(1 - \frac{1}{Q_t}\right)^{-1/\nu} C_t^{\gamma/\nu} \tag{2}$$

$$1 = \beta E_t \left\{ Q_t \frac{P_t}{P_{t+1}} \frac{C_{t+1}^{-\gamma}}{C_t^{-\gamma}} \right\} = E_t \left\{ \Lambda_{t,t+1} R_{t+1} \right\} \tag{3}$$

plus Fisher: $R_{t+1} = Q_t P_t / P_{t+1}$.

Final Goods Producer

- Produce from continuum of intermediates $i \in [0, 1]$:

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} \, di \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

- Cost minimization gives demand curve for intermediates:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t$$

- And price index:

$$P_t = \left[\int_0^1 P_t(i)^{1-\varepsilon} \, di \right]^{\frac{1}{1-\varepsilon}}$$

Intermediate Good Producers: Calvo Assumption

- Produce variety CRS with labor:

$$Y_t(i) = A_t N_t(i)$$

  - CRS will save some algebra as MC is invariant to scale. See Gali for DRS.
- Calvo (1983) pricing assumption: Each firm resets price each period with iid probability $1-\theta$.
  - By LLN, fraction that reset is $1-\theta$ and fraction constant is θ.
  - Average price duration follows geometric dist with mean duration $\frac{1}{1-\theta}$.
- Firms that adjust prices choose $P_t(i)$, $Y_t(i)$, $N_t(i)$ to maximize expected discounted profits and demand.
- Firms that do not adjust prices set output to meet demand as long as $P_t(i) > MC_t^n(i)$ (nominal MC).

Intermediate Good Producers: Calvo Assumption

- Calvo is a strong assumption!
- Is the world Calvo?
  - Literally, no.
  - But it could be a decent approximation.
- Literature on "menu cost" models where there is an inaction region due to fixed cost of changing price.
  - Initial literature: Much more flexible than Calvo, since firms that have price furthest from MC change price.
  - Recent literature: To match micro-pricing facts, need large and infrequent firm-level MC shocks, which looks close to Calvo.
  - Auclert, Rigato, Rognlie, Straub (2024): To a first order, MC equivalent to Calvo with suitably chosen adjustment frequency.
  - I (sometimes) cover in my second year class.

Price Dynamics With Calvo

- Assume symmetric model, so fraction $1-\theta$ of firms adjust to P_t^* and fraction θ keep $P_{t-1}(i)$:

$$\begin{aligned} P_t &= \left[\int_0^1 P_t(i)^{1-\varepsilon}\, di\right]^{\frac{1}{1-\varepsilon}} \\ &= \left[\theta \int_0^1 P_{t-1}(i)^{1-\varepsilon}\, di + (1-\theta)\int_0^1 P_t^{*1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}} \\ &= \left[\theta \left[\int_0^1 P_{t-1}(i)^{1-\varepsilon}\, di\right]^{\frac{1-\varepsilon}{1-\varepsilon}} + (1-\theta)\int_0^1 P_t^{*1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}} \\ &= \left[\theta P_{t-1}^{1-\varepsilon} + (1-\theta) P_t^{*1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \end{aligned} \tag{4}$$

- Price index P_t is geometric average of P_{t-1} and P_t^*.
- Recursive formulation is part of why Calvo is so tractable.

Inflation Dynamics With Calvo

- Divide by P_{t-1} to get inflation between $t-1$ and t, Π_t:

$$\Pi_t = \frac{P_t}{P_{t-1}} = \left[\theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{5}$$

- From this, we can see that Calvo price setting implies a partial adjustment mechanism:
    - If $P_t^* = P_{t-1}$, $\Pi_t = 1$.
    - If $P_t^* > P_{t-1}$, $\Pi_t > 1$ and $P_t \neq P_{t-1}$ (only asymptotically).

Optimal Intermediate Reset Price Setting

$$\max_{\{Y_{t+s|t}\}_{s=0}^{\infty}, P_t^*} E_t \left\{ \sum_{s=0}^{\infty} \theta^s \Lambda_{t,t+s}^n \left(P_t^* Y_{t+s|t} - MC_{t+s}^n Y_{t+s|t} \right) \right\} \text{ s.t}$$

$$Y_{t+s|t} = \left(\frac{P_t^*}{P_{t+s}} \right)^{-\varepsilon} Y_{t+s}$$

- Intermediate producer maximizes nominal discounted profits (could do real as well).
  - Discounting at the nominal SDF $\Lambda_{t,t+s}^n = \beta^s \frac{P_t}{P_{t+s}} \frac{C_{t+s}^{-\gamma}}{C_t^{-\gamma}}$.
  - Also discounting by prob they keep price same θ.
- Nominal Profits are:
  - P_t^* minus nominal marginal cost at time $t+s$ MC_{t+s}^n, which is taken as given.
  - Times demand at time $t+s$ given that you last set your price at t, $Y_{t+s|t}$, determined by the demand curve.
- Note: Only showing terms where P_t^* enters optimization.

Optimal Intermediate Reset Price Setting

$$E_t \left\{ \sum_{s=0}^{\infty} \theta^s \Lambda^n_{t,t+s} Y_{t+s|t} \left[\left(P^*_t - (1+\mu) MC^n_{t+s} \right) \right] \right\} = 0 \qquad (6)$$

- If $\theta = 0$, no stickiness and this collapses to flex price model:

$$P^*_t = (1+\mu) MC^n_t \text{ where } 1+\mu = \varepsilon / (1-\varepsilon)$$

- If $\theta > 0$, then the optimal reset price is a markup over a weighted average of expected future marginal costs:

$$P^*_t = (1+\mu) E_t \left\{ \sum_{s=0}^{\infty} \omega_{t,t+s} MC^n_{t+s} \right\}$$

$$\text{where } \omega_{t,t+s} = \frac{\theta^s \Lambda^n_{t,t+s} Y_{t+s} P^{\varepsilon}_{t+s}}{E_t \left\{ \sum_{k=0}^{\infty} \theta^k \Lambda^n_{t,t+k} Y_{t+k} P^{\varepsilon}_{t+k} \right\}}$$

Completing the Model

- Because of CRS, nominal marginal cost is: $MC_t^n = \frac{W_t}{Y_t(i)/N_t(i)} = \frac{W_t}{A_t}$
- Aggregate output is:

$$Y_t = \left[\int_0^1 \left[A_t N_t\left(i\right)\right]^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}} = A_t N_t \left[\int_0^1 \left(\frac{N_t\left(i\right)}{N_t}\right)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

  - Term in brackets is loss in output due to misallocation caused by price dispersion.
  - Creates welfare costs of inflation, but it is second order and drops out of log-linearization.

New Keynesian Model Equilibrium

Definition

A symmetric equilibrium is an allocation $\{C_{t+s}, N_{t+s}, Y_{t+s}\}_{s=0}^{\infty}$ and set of prices $\left\{P^*_{t+s}, P_{t+s}, W_{t+s}, Q_{t+s}\right\}_{s=0}^{\infty}$ along with exogenous processes $\{A_{t+s}, v_{t+s}\}_{s=0}^{\infty}$ such that:

1. Households optimize: Euler, labor-leisure, (money demand in background with separable utility and interest rate rule as central bank chooses Q_{t+s}, putting B_{t+s} in background too).
2. Firms optimize:
   - 2.1 Price index follows dynamic Calvo formulation.
   - 2.2 Intermediate reset prices are chosen optimally given nominal marginal cost: $MC_t^n = \frac{W_t}{Y_t(i)/N_t(i)} = \frac{W_t}{A_t}$
3. Central bank follows interest rate rule with shock v_t.
4. Labor and goods (and bond) markets clear.

Equilibrium: Nonlinear Equations

$$
\begin{aligned}
\frac{W_t}{P_t} &= \frac{\chi N_t^{\varphi}}{C_t^{-\gamma}} \\
1 &= \beta E_t \left\{ Q_t \frac{P_t}{P_{t+1}} \frac{C_{t+1}^{-\gamma}}{C_t^{-\gamma}} \right\} \\
P_t &= \left[\theta P_{t-1}^{1-\varepsilon} + (1-\theta) P_t^{*1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \\
P_t^* &= (1+\mu) E_t \left\{ \sum_{s=0}^{\infty} \frac{\theta^s \Lambda_{t,t+s}^n P_{t+s}^{\varepsilon} Y_{t+s}}{E_t \left\{ \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k}^n P_{t+k}^{\varepsilon} Y_{t+k} \right\}} \frac{W_{t+s}}{A_{t+s}} \right\} \\
Y_t &= C_t \\
Y_t &= A_t N_t \left[\int_0^1 \left(\frac{N_t(i)}{N_t} \right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}
\end{aligned}
$$

Nonlinear Equations: Intuition

- Combining MC_t^n with $Y_t = C_t$ and labor-leisure gives:

$$MC_t^n = \frac{W_t}{A_t} = \frac{\chi N_t^{\varphi} P_t / C_t^{-\gamma}}{Y_t / N_t} = \chi \frac{N_t^{1+\varphi} P_t}{Y_t^{1-\gamma}}$$

  - Approximation from assuming $Y_t = A_t N_t$ and ignoring the effect of $N_t(i)$ dispersion on Y_t (which is second order).

- Plug into P^* and use $\Lambda_{t,t+s}^n = \beta^s \frac{P_t}{P_{t+s}} \frac{C_{t+s}^{-\gamma}}{C_t^{-\gamma}}$ to obtain:

$$\begin{aligned} P_t^* &= \chi (1+\mu) E_t \left\{ \sum_{s=0}^{\infty} \frac{(\theta\beta)^s \frac{P_t}{P_{t+s}} \frac{C_{t+s}^{-\gamma}}{C_t^{-\gamma}} P_{t+s}^{\varepsilon} Y_{t+s}}{E_t \left\{ \sum_{k=0}^{\infty} (\theta\beta)^k \frac{P_t}{P_{t+k}} \frac{C_{t+k}^{-\gamma}}{C_t^{-\gamma}} P_{t+k}^{\varepsilon} Y_{t+k} \right\}} \frac{N_{t+s}^{1+\varphi} P_{t+s}}{Y_{t+s}^{1-\gamma}} \right\} \\ &\approx \chi (1+\mu) E_t \left\{ \sum_{s=0}^{\infty} \frac{(\theta\beta)^s P_{t+s}^{\varepsilon-1} N_{t+s}^{1+\varphi} P_{t+s}}{E_t \left\{ \sum_{k=0}^{\infty} (\theta\beta)^k P_{t+k}^{\varepsilon-1} Y_{t+k}^{1-\gamma} \right\}} \right\} \end{aligned}$$

- Last line from $C \approx Y$ and cancelling expectations that are $\approx$

Nonlinear Equations: Forward Looking Price Setting

$$P_t^* = \chi (1+\mu) E_t \left\{ \sum_{s=0}^{\infty} \frac{(\theta\beta)^s P_{t+s}^{\varepsilon-1} N_{t+s}^{1+\varphi} P_{t+s}}{E_t \left\{ \sum_{k=0}^{\infty} (\theta\beta)^k P_{t+k}^{\varepsilon-1} Y_{t+k}^{1-\gamma} \right\}} \right\}$$

- P_t^*, and hence inflation, *is increasing in future P_{t+k} and thus future inflation*.
    - The P_{t+s} term comes from nominal marginal costs.
    - Labor-leisure condition implies that real wage = MRS.
    - If inflation is expected to be higher, must pay higher nominal wage to hire labor, and nominal marginal costs will rise.
    - Set higher price today to cover the average discounted nominal marginal cost until can change price again.
- *Price setting is forward looking and incorporates expected future inflation.*

Nonlinear Equations: Flexible Price Equilibrium

- Shut down expected future inflation: $P_{t+s} = P_{t+s-1} \forall s > 0$.
- Consider price setting in a flexible price equilibrium:

$$\begin{aligned} P_t^* &= \chi (1+\mu) \frac{P_t N_t^{1+\varphi}}{Y_t^{1-\gamma}} \\ Y_t^{1-\gamma} &= \chi (1+\mu) N_t^{1+\varphi} \end{aligned}$$

- If in flex price equilibrium, on average expect to be in future.
  - Are higher order terms from uncertainty in the expectation, but neglect for first-order intuition.
- With no expected future inflation:

$$P_t^* \approx P_t \times E_t \left\{ \sum_{s=0}^{\infty} \frac{(\theta\beta)^s Y_{t+s}^{1-\gamma}}{\sum_{k=0}^{\infty} (\theta\beta)^k Y_{t+k}^{1-\gamma}} \right\} = P_t$$

Nonlinear Equations: Flexible Price Equilibrium

$$P_t^* \approx P_t \times E_t \left\{ \sum_{s=0}^{\infty} \frac{(\theta\beta)^s \, Y_{t+s}^{1-\gamma}}{\sum_{k=0}^{\infty} (\theta\beta)^k \, Y_{t+k}^{1-\gamma}} \right\} = P_t$$

- If at flexible price level of output and do not expect future inflation, no inflation today because resetters choose to set flexible price.
  - In flexible price equilibrium, average markup is at desired level.
  - And there is no expected inflation.
  - So resetters set their markup equal to average markup and there is no inflation.

Nonlinear Equations: Phillips Curve

- What if not at flex price equilibrium and no expected future inflation?
- Consider $Y_t > Y_t^{flex}$.
  - Then $Y_t \approx A_t N_t$ so $N_t^{1+\varphi}$ rises more than $Y_t^{1-\gamma}$.
  - This causes the numerator to rise, and with no expected future inflation, we have inflation today:

$$P_t^* \approx P_t \chi (1+\mu) E_t \left\{ \sum_{s=0}^{\infty} \frac{(\theta\beta)^s N_{t+s}^{1+\varphi}}{\sum_{k=0}^{\infty} (\theta\beta)^k Y_{t+k}^{1-\gamma}} \right\} > P_t$$

  - If output is unexpectedly above its flexible-price level, labor must be higher, causing nominal wage to be higher, and nominal marginal costs to be higher. Resetters raise prices to cover higher nominal marginal cost.

Nonlinear Equations: Phillips Curve

- Putting everything together, the NK model features an *expectations-augmented Phillips curve*.
  - Because price setting is forward looking, Phillips curve is going to be *expectation-augmented*.
  - Output-inflation relationship relates inflation to deviations from flex price output (called the *natural level of output*).
    - At natural level of output with no expected inflation, resetters choose price equal to flexible price, resulting in no inflation.
    - If output deviates, that is if the *output gap* $\tilde{Y}_t = Y_t - Y_t^{flex}$ is nonzero, the extra output will bid up nominal wages and move marginal costs, causing inflation in response to deviations of output from its flexible level.
- NK model is sensitive to Lucas Critique, unlike "old" Keynesian models.
  - Central bank cannot regularly exploit output-inflation trade-off.

Nonlinear Equations: Central Bank and Labor Wedge

- Can now introduce central bank rule that responds to inflation and output gap:

$$\hat{i}_t = \phi_\pi \hat{\pi}_t + \phi_y \tilde{y}_t + v_t$$

- What is the labor wedge?
  - Note that for firm with markup $\mu_t(i)$,

$$1 + \mu_t(i) = \frac{P_t(i) / P_t}{MC_t}$$

  - But on average, $P_t(i) / P_t = 1$ so

$$1 + \mu_t = 1/MC_t = \frac{Y_t/N_t}{W_t/P_t} = \frac{MPL_t}{MRS_t}$$

  - So the labor wedge is the average markup, which is countercyclical.

Log Linearization Strategy

- Phillips curve should be function of output gap, so want to write whole model as function of output gap.
- Strategy:
  1. Log-Linearize Model *Around Zero-Inflation Steady State*
     1.1 AD Block: Euler Equation $\Rightarrow$ Dynamic IS Curve
     1.2 AS Block: Pricing and MC Equations $\Rightarrow$ NK Phillips Curve
     1.3 Central Bank Monetary Rule
  2. Log-Linearize Flex Price Equilibrium
  3. Difference To Get Equilibrium In Terms of Output Gap
- Today: Only through AS block. AD block and 3 equation model next class.

Log Linearization: The Aggregate Supply Block

$$\frac{W_t}{P_t} = \frac{\chi N_t^{\varphi}}{C_t^{-\gamma}}$$

$$P_t = \left[\theta P_{t-1}^{1-\varepsilon} + (1-\theta) P_t^{*1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$

$$P_t^* = (1+\mu) E_t \left\{ \sum_{s=0}^{\infty} \frac{\theta^s \Lambda_{t,t+s}^n P_{t+s}^{\varepsilon} Y_{t+s}}{E_t \left\{ \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k}^n P_{t+k}^{\varepsilon} Y_{t+k} \right\}} \frac{W_{t+s}}{Y_{t+s}/N_{t+s}} \right\}$$

$$Y_t = A_t N_t \left[\int_0^1 \left(\frac{N_t(i)}{N_t} \right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

- Labor-Leisure and production function are standard:

$$\hat{w}_t - \hat{p}_t = \varphi \hat{n}_t + \gamma \hat{c}_t$$
$$\hat{y}_t = \hat{a}_t + \hat{n}_t$$

Log Linearization: Inflation and Reset Prices

- Key trick: Zero inflation steady state.
  - I will skip a lot of painful log-linearization.
- The price index can be log-linearized to get

$$\hat{p}_t = \theta \hat{p}_{t-1} + (1-\theta)\,\hat{p}_t^*$$

  - Equivalently written in terms of inflation:

$$\hat{\pi}_t = (1-\theta)\left(\hat{p}_t^* - \hat{p}_{t-1}\right)$$

Log Linearization: Reset Prices

- The reset price can be log-linearized as:

$$
\begin{aligned}
a\hat{p}_t^* &= (1-\beta\theta) E_t \left\{ \sum_{s=0}^{\infty} (\beta\theta)^s \left(\hat{mc}_{t+s}^n \right) \right\} \\
&= (1-\beta\theta) E_t \left\{ \sum_{s=0}^{\infty} (\beta\theta)^s \left(\hat{mc}_{t+s} + \hat{p}_{t+s} \right) \right\}
\end{aligned}
$$

  - $\hat{mc}_t = \log MC_t - \log MC^{ss}$, where $mc_t \equiv \log MC_t$. Note that $\log MC^{ss} = -\mu$ in the zero inflation steady state.
  - Consequently, $\hat{mc}_{t+s} = mc_{t+s} + \mu$.

- We can write this recursively as:

$$
\hat{p}_t^* = (1-\beta\theta)\left(\hat{mc}_t + \hat{p}_t \right) + \beta\theta E_t \left\{ \hat{p}_{t+1}^* \right\}
$$

Log Linearization: Phillips Curve

- Subtract $\hat{p}_{t-1}$:

$$\hat{p}_t^* - \hat{p}_{t-1} = (1 - \beta\theta)\, \hat{mc}_t + \hat{\pi}_t + \beta\theta E_t \left\{ \hat{p}_{t+1}^* - \hat{p}_t \right\}$$

- Plug into $\hat{\pi}_t = (1-\theta)\left(\hat{p}_t^* - \hat{p}_{t-1}\right)$ to get an expectations-augmented Phillips curve:

$$\hat{\pi}_t = \lambda \hat{mc}_t + \beta E_t \left\{ \hat{\pi}_{t+1} \right\} \text{ where } \lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}$$

- Inflation is equal to expected future inflation plus the deviation of marginal cost from its steady state level.
  - Expected inflation: Forward looking price setters choose higher prices now if inflation is expected to be high, as nominal marginal costs will rise.
  - Slope λ is increasing in Calvo reset prob $1-\theta$.
    More resetters $\Rightarrow$ faster price response to MC change.

Log Linearization: Phillips Curve

- Inflation is equal to expected future inflation plus the deviation of marginal cost from its steady state level.
  - Two ways to think about marginal cost deviation:

  1. Set higher prices to cover higher marginal cost.
  2. When marginal costs are above desired level, markups are below desired level. Inflation as firms hike markup back to desired level. (In fact, $\hat{mc}_t = -\hat{\mu}_t$).

- Iterating forward,

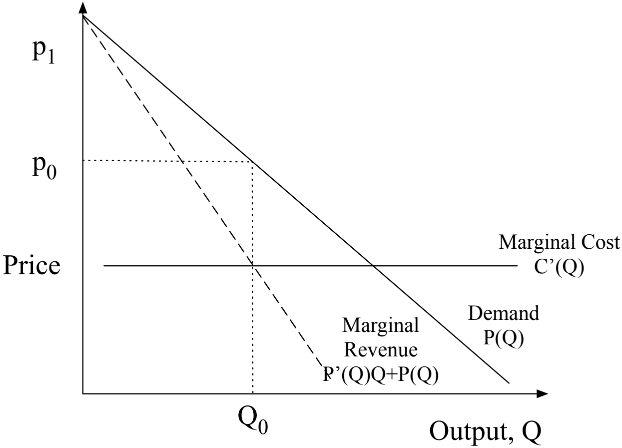
$$\hat{\pi}_t = \lambda E_t \left\{ \sum_{s=0}^{\infty} \beta^s \hat{mc}_{t+s} \right\}$$

.

  - Inflation is the PDV of future marginal cost / markup deviations from steady state.

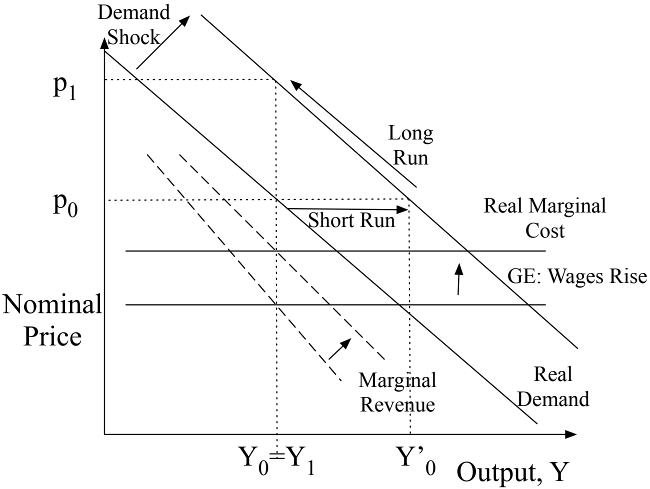
Intuition: Back to Monopoly Diagram

- If expect MC to rise due to inflation but price to be stuck, raise price today to get markup right "on average."



Intuition: Back to Monopoly Diagram

- With Calvo, demand shock has similar effect to fixed short run price diagram with geometrically declining fraction of firms.



Log Linearization: Real Marginal Costs

- Combine labor-leisure, production function $\hat{n}_t = \hat{y}_t - \hat{a}_t$, and $\hat{c}_t = \hat{y}_t$:

$$\hat{w}_t - \hat{p}_t = (\gamma + \varphi)\,\hat{y}_t - \varphi\hat{a}_t$$

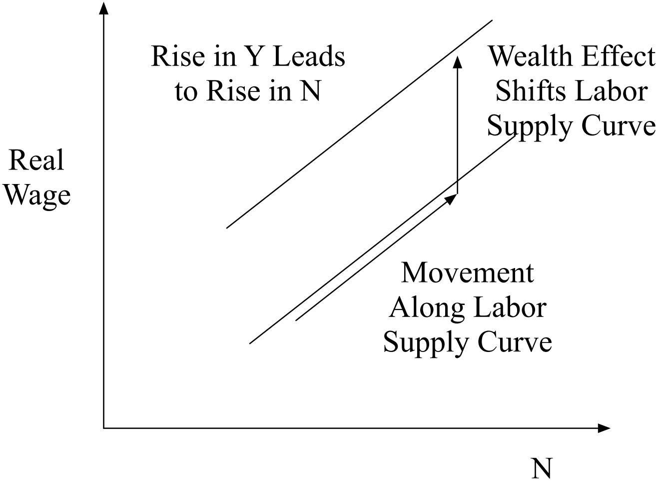
- Consequently

$$\begin{aligned}\hat{mc}_t &= \hat{w}_t - \hat{p}_t - \hat{a}_t \\ &= (\gamma + \varphi)\,\hat{y}_t - (1 + \varphi)\,\hat{a}_t\end{aligned}$$

- Intuition: In labor-only model, MC determined by real wage per unit of output. When output rises above steady state, move up labor supply curve pushing up real wages and MC.
  - Effect 1: Moving up labor supply curve with Frisch $1/\varphi$.
  - Effect 2: Shift up in labor supply curve due to wealth effect. Strength of shift related to IES $1/\gamma$.
  - Tech improvement $\Rightarrow$ hire less labor so less movement up labor supply curve and direct effect on MC due to per unit output.

Real MC Intuition

- I like to draw a labor market equilibrium.



Log Linearization: Flexible Price Equilibrium

$$
\begin{aligned}
Y_t^{flex} &= A_t N_t^{flex} \\
\frac{W_t^{flex}}{P_t^{flex}} &= \frac{A_t}{1+\mu} \\
\frac{W_t^{flex}}{P_t^{flex}} &= \frac{\chi \left(N_t^{flex}\right)^{\varphi}}{\left(C_t^{flex}\right)^{-\gamma}} \\
Y_t^{flex} &= C_t^{flex}
\end{aligned}
$$

- Combine to get:

$$
\begin{aligned}
A_t^{1+\varphi} &= \chi (1+\mu) \left(Y_t^{flex}\right)^{\gamma+\varphi} \\
(\gamma+\varphi)\, \hat{y}_t^{flex} &= (1+\varphi)\, \hat{a}_t
\end{aligned}
$$

Real Marginal Costs in Terms of Output Gap

- Combine:

$$\begin{aligned} \hat{mc}_t &= (\gamma + \varphi)\, \hat{y}_t - (1 + \varphi)\, \hat{a}_t \\ (\gamma + \varphi)\, \hat{y}_t^{flex} &= (1 + \varphi)\, \hat{a}_t \end{aligned}$$

  to write real marginal costs in terms of output gap $\tilde{y}_t$:

$$\hat{mc}_t = (\gamma + \varphi) \left(\hat{y}_t - \hat{y}_t^{flex} \right) = (\gamma + \varphi)\, \tilde{y}_t$$

- Real marginal costs go up (and markups go down) when the output gap is high.
  - To produce more than under flex prices, markup must be lower.
  - Marginal costs high because need to hire more workers, bidding up real wage.
  - Stronger when IES and labor supply elasticity are low due to labor market equilibrium intuition.

The New Keynesian Phillips Curve

- Plug back into the Phillips curve $\hat{\pi}_t = \lambda \hat{mc}_t + \beta E_t \{\hat{\pi}_{t+1}\}$:

$$\hat{\pi}_t = \kappa \tilde{y}_t + \beta E_t \{\hat{\pi}_{t+1}\} \text{ where } \kappa = \lambda(\gamma + \varphi)$$

  - This is the *New Keynesian Phillips Curve*: an expectations augmented Phillips curve written in terms of the output gap.

- Solving forward,

$$\hat{\pi}_t = \kappa E_t \sum_{s=0}^{\infty} \beta^s \tilde{y}_{t+s}$$

  - Inflation is an increasing function of future output gaps.
  - Output gap high $\Rightarrow$ marginal cost high and markups low $\Rightarrow$ raise markups.

- Next class: AD block, boil down to three-equation model, and then solve and critique NK model.